ON THE SUFFICIENT CONDITIONS OF STABILITY IN THE THEORY OF A HORIZONTAL GYROCOMPASS

(O DOSTATOCHNYKH USLOVIAKH USTOICHIVOSTI V TEORII GIROGORIZONTKOMPASS)

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This note develops and sharpens the results obtained in [1].

If the moment generated by a spring connecting two gyroscopes satisfies [2] the condition

$$N(\varepsilon) = -\frac{4B'^2}{mlR}\cos\varepsilon\sin\varepsilon \tag{1}$$

and if otherwise the assumptions are the same as those stated in [1], then the energy integral has the form

$$V \equiv \frac{1}{2}Ap^2 + \frac{1}{2}Bq^2 + \frac{1}{2}Cr^2 + I\dot{\varepsilon}^2 - \frac{1}{2}\frac{4B'^2}{mlR}\sin^2\varepsilon - (F - mv\omega)\,l\psi_3 - mvl\Omega\vartheta_3 -$$
$$- \Omega\,[Ap\psi_1 + (Bq + H)\psi_2 + Cr\psi_3] - \omega\,[Ap\vartheta_1 + (Bq + H)\vartheta_2 + Cr\vartheta_3] = C_1 \left(\omega = \frac{v}{R}\right) \tag{2}$$

Here $I\dot{\varepsilon}^2$ is the kinetic energy of the gyroscopes with their casings as they rotate about the axes of the casings. This energy has been neglected in [1].

The position of equilibrium of the system occurs at the following values of the coordinates:

$$\alpha = 0, \qquad \beta = \beta^*, \qquad \gamma = 0, \qquad \delta = \delta^* \tag{3}$$

and β^* and δ^* satisfy the equations

$$(C - B)[{}^1/_2\,(\Omega^2 - \omega^2)\sin 2\beta^* + \omega\Omega\cos 2\beta^*] - 2B'\cos(\varepsilon_0 + \delta^*)(\Omega\cos\beta^* - \omega\sin\beta^*) =$$
$$= -(F - mv\omega)\,l\sin\beta^* - mvl\Omega\cos\beta^*$$
$$-(\omega\cos\beta^* + \Omega\sin\beta^*)\,2B'\sin(\varepsilon_0 + \delta^*) = N(\varepsilon_0 + \delta^*) \tag{4}$$

Here ε_0 is a particular value of ε which is the angle of separation of the gyroscopes satisfying the relation

$$\varepsilon_0 = \cos^{-1}\frac{mlv}{2B'} \tag{5}$$

If the motion described by the equations (3) is unperturbed, then we can obtain for it [1] the sufficient conditions of stability in the form

$$c_{11} > 0, \quad c_{22} > 0, \qquad c_{11}c_{33} - c_{13}^2 > 0, \qquad c_{22}c_{44} - c_{24}^2 > 0 \tag{6}$$

Here

$$\begin{aligned}
c_{11} &= {}^1\!/_2\omega\,\{-mlR\,\Omega\sin\beta^* - A\omega + [B(\omega\cos\beta^* + \Omega\sin\beta^*) + \\
&\quad + 2B'\cos(\varepsilon_0+\delta^*)]\cos\beta^* + C(\omega\sin\beta^* - \Omega\cos\beta^*)\sin\beta^*\} \\
c_{22} &= {}^1\!/_2\,\{(C-B)\,[(\Omega\cos\beta^* - \omega\sin\beta^*)^2 - (\Omega\sin\beta^* + \omega\cos\beta^*)^2] + [(F-mv\omega)\,l + \\
&\quad + \omega\,2B'\cos(\varepsilon_0+\delta^*)]\cos\beta^* - \Omega\,[mvl - 2B'\cos(\varepsilon_0+\delta^*)]\sin\beta^*\} \\
c_{33} &= {}^1\!/_2\,[(C-A)(\Omega\cos\beta^* - \omega\sin\beta^*)^2 + (F-mv\omega)\,l\cos\beta^* - mvl\Omega\sin\beta^*] \\
c_{44} &= {}^1\!/_2\,[-(4B'^2/mlR)\cos 2(\varepsilon_0+\delta^*) + 2B'\cos(\varepsilon_0+\delta^*)(\Omega\sin\beta^* + \omega\cos\beta^*)] \\
c_{13} &= {}^1\!/_2\omega\,[(C-A)(\omega\sin\beta^* - \Omega\cos\beta^*) - mlR\,\Omega] \\
c_{24} &= {}^1\!/_2\,2B'\sin(\varepsilon_0+\delta^*)(\Omega\cos\beta^* - \omega\sin\beta^*)
\end{aligned} \tag{7}$$

Let us mention that the equations (4) have a solution $\beta^* = 0$, if $N(\varepsilon)$ satisfies [3] the condition

$$N(\varepsilon) = -\frac{4B'^2}{mlR(1+\chi)}\cos\varepsilon\sin\varepsilon \qquad \left(\chi = \frac{(C-B)}{mlR}\right) \tag{8}$$

Here the value of δ^* is determined by the equation

$$2B'\cos(\varepsilon_0+\delta^*) = mlv(1+\chi) \tag{9}$$

The integral (2) retains its form except for the coefficient of $\sin^2\varepsilon$, where in the denominator appears an additional factor $(1-\chi)$.

In this case the inequalities (6) assume a simple form

$$\begin{gathered}
mlv\left[1+\frac{(C-A)}{mlR}\right] > 0, \qquad Fl\left[1+\chi\left(\frac{\Omega}{\nu}\right)^2\right] > 0 \\
F - mv\omega - mR\,\Omega^2 > 0, \qquad F - mR\,\Omega^2 > 0
\end{gathered} \qquad \left(\nu = \sqrt{\frac{g}{R}}\right) \tag{10}$$

The sufficient conditions of stability (6) or (10) do permit degenerations similar to those shown in [1].

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